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Efficient Modeling of Multi-Component Maintenance

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Abstract

Prediction of life cycle costs requires estimation of the expected frequency of relevant events (such as component failures, forced outages, repairs, replacements, inspections, etc.). For modern engineering systems that consist of thousands of components, a component-based approach to the system-level modeling promises not only computational savings, but, even more importantly, the reduction of the modeling complexity. One of the challenges in creating accurate component-based models is to take into account coupling among individual maintenance schedules for each component. To this end, a recently developed method is advocated for compact representation of the aggregate effects of competing risks from multiple components (e.g., in the context of opportunistic maintenance and induced failures). It is shown that a single Weibull distribution can successfully represent those aggregate effects as long as the inspection/replacement intervals are relatively conservative. In order to quantify the accuracy of this method numerical integration based on finite-differences is employed for age-based replacement maintenance policies. The convergence of the finite-difference scheme is also assessed and compared to the truncated mean time life approximation as a function of time horizon. Finally, an example of implementation of the advocated component-based modeling of multicomponent system using stochastic Petri nets with aging tokens is presented and discussed in the context of alternative state-space representation.

Introduction

Unlike maintenance policies for a small number of units that are relatively well understood, the challenges of developing optimal policies for complex systems consisting of thousands of individual entities (i.e., line-replaceable units or LRUs) remain quite formidable. In order to model maintenance policies of a system, some state representation as the description of the dynamic of the corresponding state transition is required. For a small system, a custom Monte Carlo simulation maintenance models can be easily developed, but using standardized graphical representation provides advantages for creating larger models, in particular for verification purposes. Fuurthermore, local or component-based representation of the state space usually scale better with the size of the problem: instead of each state representing the system as a whole (or globally, as in Markov chains), states of individual components are described along with their interactions, so that the system state can be inferred from its component states (rather then described explicitly). This is the essence of Stochastic Petri Nets (SPNs) [1],[2], where individual components are denoted with small circles (called tokens) and their transitions between states (called places) are denoted with solid rectangles. The majority of the implementations of SPNs rely on automated translation of the local-representation into the global (Markovian) model [3]. However, the use of Monte-Carlo simulation is also a viable alternative[4] as it allows to avoid any restrictions on the type of distributions for the delays in state transitions. In traditional SPNs tokens are indistinguishable, but in so-called colored Petri nets [5], tokens have unique identities (labels), so an alternative interpretation of firing facilitates the preservation of the information about the

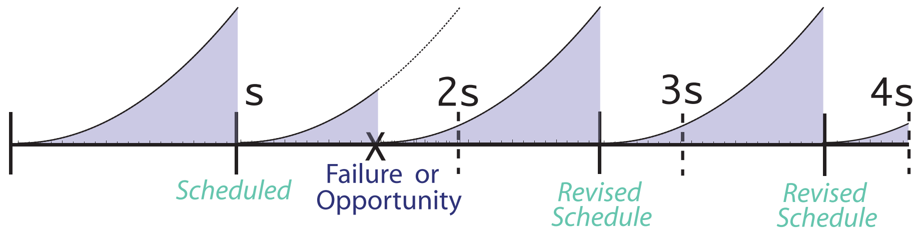

Figure 1: Age-based replacement maintenance schematics

system's past states: rather than considering removing a token from the transition's input place and depositing a (different) token to the output place as two disjoint actions, one can unite these two actions into a single action of moving the same token from an input place to the output place. Memory (continuously changing labels) can be assigned to tokens (the result is "aging tokens" [6]). Such tokens can move freely throughout the Petri net without losing their memory. Firing delays for timed transitions can be interpreted by associating backward clocks: the clock starts when transition gets enabled, and once the clock reaches zero, the firing takes place. In standard SPNs, this clock is associated solely with the transition, and if more than one token is present in the input place, the token to be fired is selected at random. With aging tokens, a clock is associated with a token-transition pair, which allows several clocks to run simultaneously for a single transition, and often results in a more compact model. As shown next.

Numerical integration of age-based replacement

Let us consider the example of a system consisting of n identical components that is subject to age-based replacement [7]: all parts are replaced after a specified interval s. A failed component causes a total renewal of the system: all components (not only the failed one) are replaced with brand new ones; furthermore, we assume that the replacements occur instantaneously. Note that failures can shift the original schedule of replacements (see Figure 1). This is the case of perfect coordination of the components' behavior, as their ages are always in sync. From the system perspective, this is a three-state semi-Markov process, with the states and transitions depicted in Figure 2: either scheduled maintenance takes place and all n components are replaced, or one of the components fails, while the rest (*i.e.,* $n-1$ components) are replaced as a result of opportunistic maintenance. Here the outputs of interest are the expected numbers of scheduled replacements and failures. The number of opportunistic maintenance actions for a given component is simply the sum of the failures for all other components. If all the transitions in Figure 2 follow exponential distributions, the resulting process is Markov [8]. Otherwise, *e.g.,* when failures follow Wiebull distribution with $\beta \neq 1$, the resulting process is only semi-Markov. There are no closed-form solution existing for finite time horizons, but two numerical options exist: numerical integration of the corresponding renewal integral equations, or discrete-event simulation. Both options can be useful, as finite-difference solutions can provide accurate solutions for smaller and simpler problems that can serve as bench marks, while simulations can be used for larger-scale problems.

Renewal equations

For a single component with no scheduled maintenance (when the part is replaced only upon failure) the corresponding renewal process is well studied. For our purposes the following form of the corresponding integral equation is useful:

$$m(t) = f(t) + \int_0^t m(\tau) f(t-\tau) d\tau \qquad (1)$$

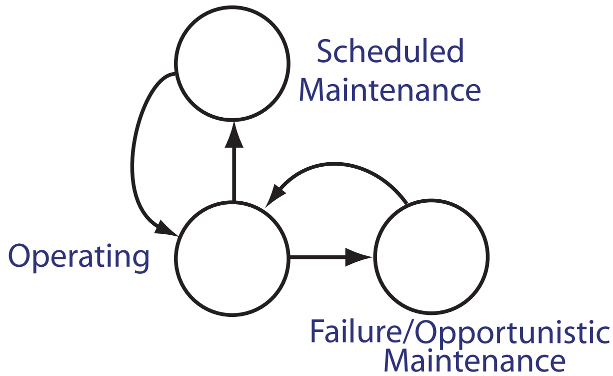

Figure 2: Global (system) state space representation of a simple system with identical components

Here the renewal density $m(t) = \frac{dM(t)}{dt}$, $M(t)$ is the expected number of renewals, and $f(t)$ is corresponding to the failure probability density function [7, 9]. Efficient methods for the numerical solution of renewal equations exist using, for example, finite differences [10, 11]. The form of Eq. 1 provides a natural interpretation that is amenable to generalization: renewal at time t can occur either due to the first failure at that time $f(t)$, or due to repeated failure, where the previous renewal took place at time τ with the corresponding renewal density $m(\tau)$, and the chances of the failure $f(t-\tau)$, for that renewal time. When scheduled replacements are introduced at time s for the first cycle $0 < t < s$, Eq. 1 remains unchanged for the expected density of failures $u(t) = m(t)$, while the expected density of scheduled replacements is $w(t) = 0$. Noting that $R(s) = 1 - F(s)$ represents the chances that no failures will occur throughout the segment s, we conclude that in the vicinity of the first scheduled replacement $t = s$, the scheduled replacement density acts as a Dirac delta function $w(t) = R(s)\delta(t-s)$. For $s \leq t$, the total renewal density is given as $m(t) = u(t) + w(t)$, and the following system of equations can be obtained:

$$w(t) = R(s)m(t-s) = R(s)\left[u(t-s) + w(t-s)\right] \tag{2}$$

$$u(t) = \int_{t-s}^{t} m(\tau)f(t-\tau)d\tau = \int_{t-s}^{t} \left[u(\tau) + w(\tau)\right] f(t-\tau)d\tau \tag{3}$$

In general, for n distinct components that can all cause renewal, we can introduce $u_i(t)$, $i \ldots n$, and the corresponding equations have the following form:

$$w(t) = R(s)m(t-s) = R(s)\left[w(t-s) + \sum_{i=1}^{n} u_i(t-s)\right] \tag{4}$$

$$u_i(t) = \int_{t-s}^{t} m(\tau)f_i(t-\tau)d\tau = \int_{t-s}^{t} \left[w(\tau) + \sum_{i=1}^{n} u_i(\tau)\right] f_i(t-\tau)d\tau \tag{5}$$

Solving these equations using finite differences can lead to highly accurate results as long as the selected time step is small enough. If Figure 3 convergence of the the solution of age-based replacement is shown for predicted number of failures and scheduled replacements. Failures follow Weibull with the shape and , scale parameters $\beta = 3$ and $\theta = 1$, respectively. Here the reference solution is 5000 steps. As another reference point it is instructive to evaluate the discrepancy with the mean-life approximation as a function of the time horizon: Figure 4, shows the discrepancy (numerical integration is conducted with 1000 steps). One can see the the numerical integration remains stable for large time horizon and converges to the mean-life approximation as expected.

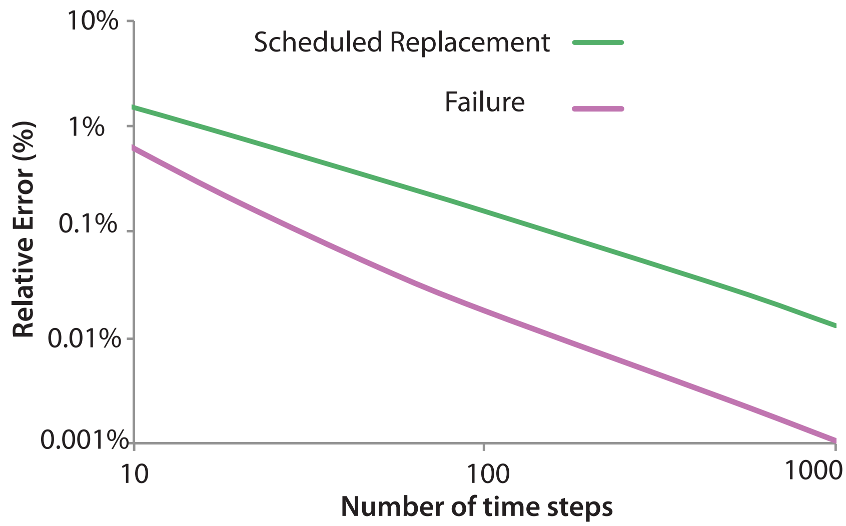

Figure 3: Convergence of the solution of age-based replacement: $s = 0.5$, $T = 2$ with respect to number of time steps per s. Weibull shape $\beta = 3$, scale $\theta = 1$

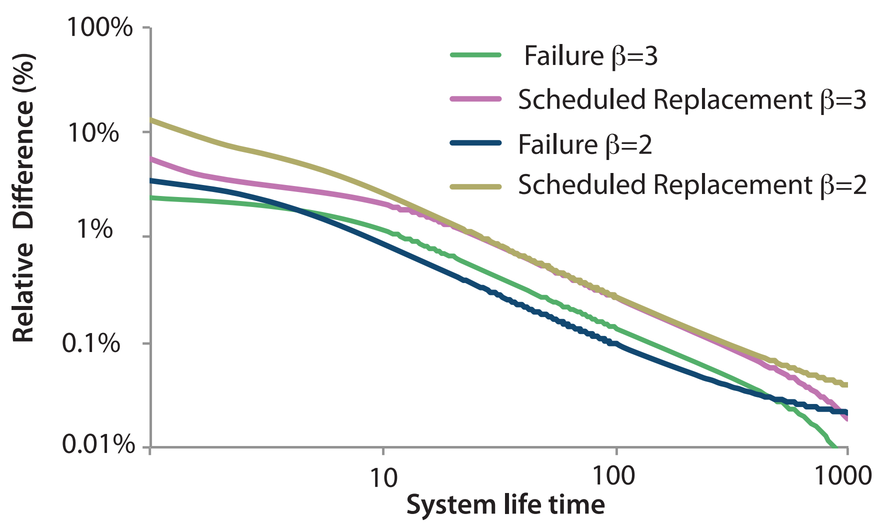

Figure 4: Comparing finite-difference integration with the truncated mean-life approximation Weibull shape $\beta = 2, 3$, scale $\theta = 1$

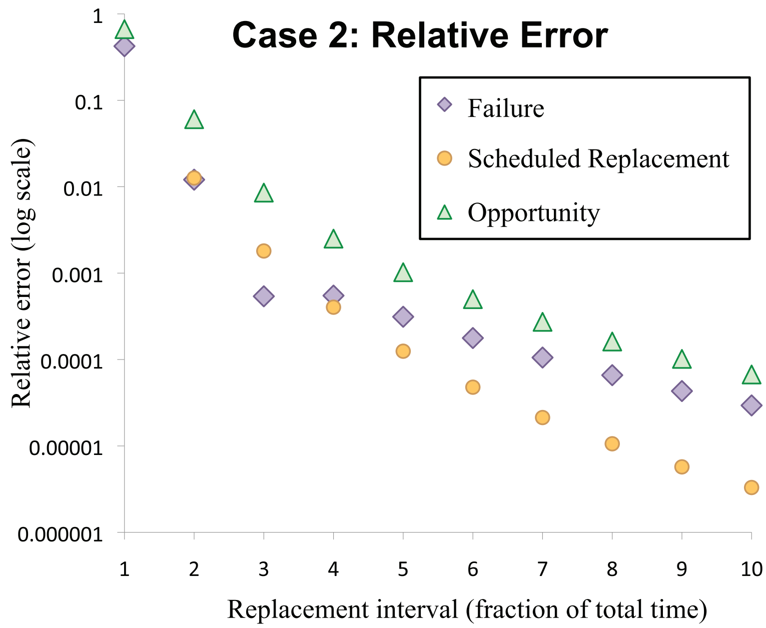

Figure 5: Relative errors for Case 2 as a function of the number of replacement intervals per time $T = 5$

Verification of component-based models for age-based replacement

In the recent work by the authors [12] a new procedure for selecting parameters of Weibull distribution has been developed that is based on matching so-called "winning ratio" of the combined distributions. Numerical integration was used to assess the accuracy of this approximation. For example, let us consider a system where two components are identical, but the third component is distinct: $\beta_1 = \beta_2 = 4$, $\theta_1 = \theta_2 = 3$; $\beta_3 = 2$, $\theta_3 = 5$. Then the opportunity for the third component follows Weibull distribution with $\beta_{1o} = 4$ and $\theta_{1o} \approx 2.5227$. For the other two components we use the developed method, and by then compare the results for the first component by evaluating the coupled model, subtracting the results for the third component, and dividing the results by two (since the first two components are identical). For the total time $T = 5$, the relative errors are shown in Fig. 5, demonstrating that for a broad range of replacement intervals, $s < T/2$ are not exceeding 1%, which for most of the application is sufficient. The natural question arises regarding the relative importance of the shape selection, as one can envision the possibility that for any shape parameter, the accuracy will be reasonable as long as the scale-matching is performed, and perhaps other shape parameters might provide even better accuracy. The results shown in Figure 6 directly address this question by varying the shape parameter parametrically for a fixed replacement interval $s = T/5 = 1$, and matching the scale for every shape. One can observe that the shape selected based on the proposed procedure $\beta \approx 4.2$ is indeed quite close to optimal, and the sensitivity with respect to the shape parameter is not trivial. We note that the perfect match would require that all three curves in Figure 6 intersect in a single point with the zero ordinate. It is also interesting to observe a non-linear dependence of the failure prediction errors (as exponential distribution can provide a smaller error that $\beta = 1.5$.)

Further detailed are provided in [12].

SPN models for multi-unit maintenance policies

Next, let us consider an example of a multi-unit system and develop corresponding SPN models. The system consists of ten components and two maintenance levels: "minor" and "major" (every other maintenance

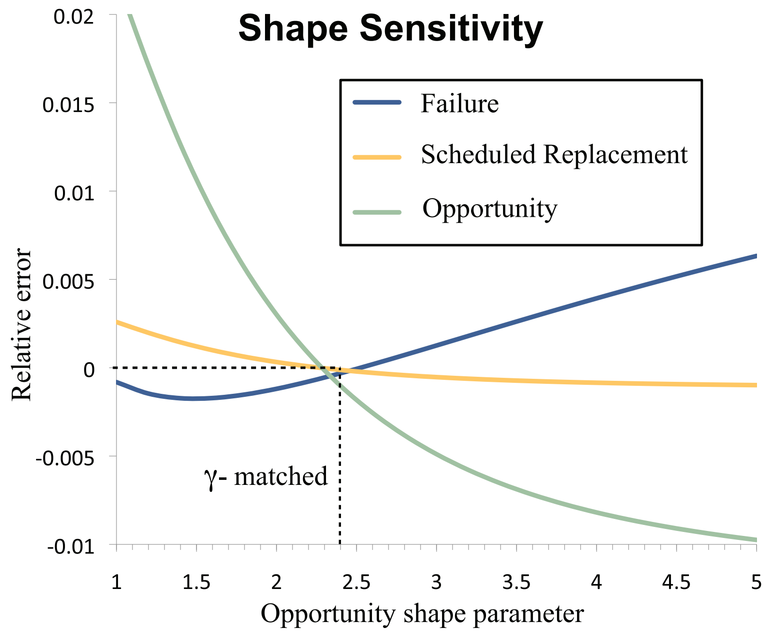

Figure 6: Sensitivity of the errors for $s = T/5 = 1$ for Case 2 as a function of shape parameter β for opportunity. Dashed lines represent the shape selected in accordance with the matched γ ratio.

is major. The first five components (denoted as components of the first type) are replaced during every minor and major maintenance, while the other five components are replaced only during the major maintenance. The failure of the first type component leads to an opportunistic minor maintenance (and therefore shifting forward the overall schedule) while the failure of the second type component leads to the major maintenance. First let us consider a fully coupled model, as depicted in Fig 7. We note that advantage of the colors are introduced here, as components of each type can have different failure distributions (denoted by different colors). The right portion of the model represents the scheduler of the maintenance - the corresponding token can have two colors 0 - meaning it is in the first cycle (before the major maintenance) and 1 during the second cycle (before the major maintenance). When the first cycle ends, and the scheduler token moves to place “minor” it cannot move further down to place “major” as the corresponding transition does not have a matching policy for color 0, so the token returns to the scheduler position with the color changed to 1, so that during the next cycle it can move to the major place. We note that here only enablers, and immediate transitions have to be timed appropriately to allow enough time to trigger appropriate actions. Fig 8 depicts a similar model for component of type 1 (the model for component 2 is similar, with the only difference stemming from the fact that the failure triggers major maintenance, so that the the scheduler token is moved to the major maintenance place). One can note that component level models only involves one component at a time. We also note that the age of the scheduler token is preserved after the minor maintenance to appropriately represent major opportunity distribution that is calculated as described in [12].

Summary & Conclusions

In this paper a component-based approach to modeling of maintenance processes of complex systems is presented. Stochastic Petri nets with aging tokens are employed to represent component-based state space. Opportunistic maintenance that couples behavior of individual components (LRUs) of the modeled system is represented by means of a single Weibull distribution, and the accuracy of this representation

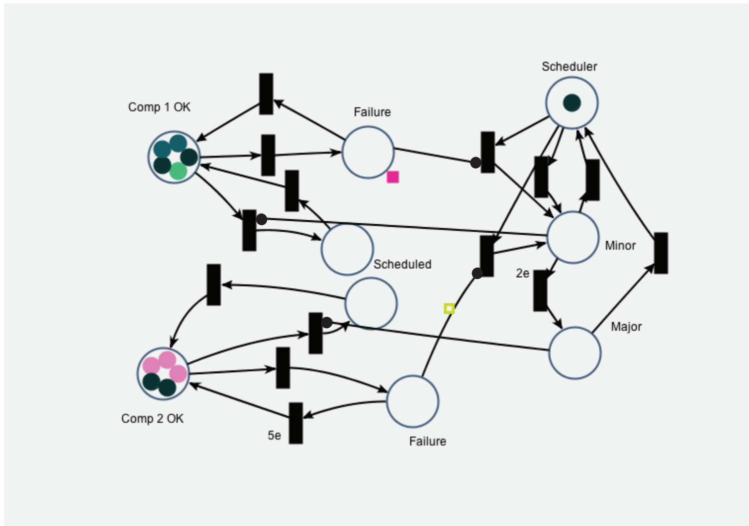

Figure 7: SPN model of age-based replacement for a system with two maintenance levels.

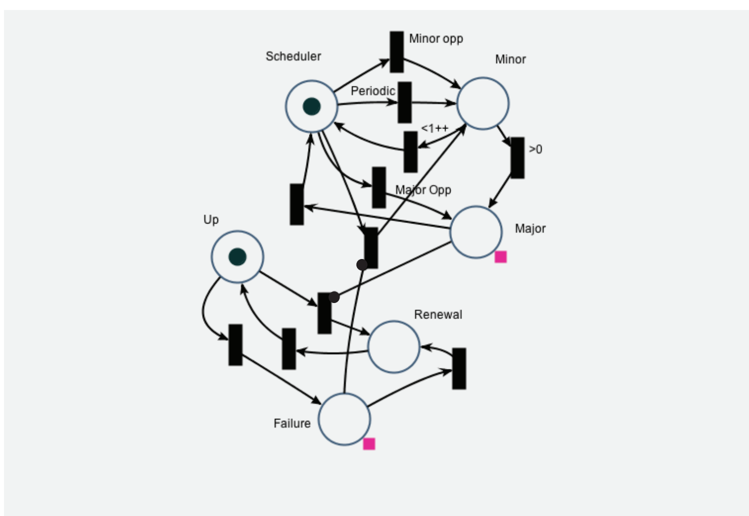

Figure 8: Component level SPN model of age-based replacement for a system with two maintenance levels (component of type 1)

is assessed for age-based replacement by means of numerical integration of the corresponding renewal equations. The convergence of this numerical integration is also assessed.

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